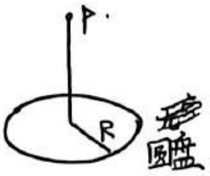
实验名称：________ 姓名：________ 学号：________

· 库仑定理. $F_{12}=\frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1q_2}{r_{12}^2}\hat{e}_{12}$ 电偶极矩：$\vec{p}=q\vec{d}$ $\ominus\xrightarrow[\vec{p}]{\vec{d}}\oplus$ $\vec{\tau}=\vec{p}\times\vec{E}$

联

$\vec{\tau}=\vec{\mu}\times\vec{B}$



求P处场强：①、求E积分

②、$E_z=-\frac{\partial V}{\partial z}$.

~~无~~ 圆盘

$$E_z=\frac{\sigma}{2\varepsilon_0}\left(1-\frac{1}{\sqrt{1+(R/z)^2}}\right)$$

· 通量 Flux：$\Phi_E=\oint\vec{E}\cdot d\vec{A}$

i)、均匀电球壳 外 $E=\frac{Q}{4\pi r^2\varepsilon_0}$ 内：0

ii)、带电球体（实心）

绝缘 → 与万有引力类似

$E=\begin{cases}\frac{Qr}{4\pi\varepsilon_0R^3} & r<R\\ \frac{Q}{4\pi\varepsilon_0r^2}, & r>R.\end{cases}$

导体 → 电荷仅分布外表面 内部E=0 表面 $E=\frac{\sigma}{\varepsilon_0}$

iii)、均匀带电平面 取高斯面. $\Rightarrow E=\frac{\sigma}{2\varepsilon_0}$

· Gauss' Law：$\Phi_e=\oint\vec{E}\cdot d\vec{A}=\frac{q}{\varepsilon_0}$ （真空，无介质）.

$\boxed{电介质}$ 无数个 $\vec{P}$ 组成 $\vec{P}$ 极化位移矢量.

$\oint\vec{D}\cdot d\vec{A}=\sum q_0$ $\vec{D}=k_e\varepsilon_0\vec{E}=\varepsilon_0\vec{E}+\vec{P}$

$\oint\vec{P}\cdot d\vec{A}=\sum_{out}q'=-\sum_{in}q'$

· 环路定理（静电场）.

$\oint\vec{E}\cdot d\vec{l}=0$ 静电场 $\xrightarrow{扩}$ 感应电场（变化磁场引发）

$\oint\vec{E}_{ind}\cdot d\vec{l}=\varepsilon=-\frac{d\Phi_B}{dt}$

$\Downarrow$

$\oint\vec{E}\cdot d\vec{l}=-\frac{d\Phi_B}{dt}=-\iint\frac{\partial\vec{B}}{\partial t}\cdot d\vec{A}$ Stokes公式

$\oint\nabla\times\vec{E}\cdot d\vec{A}$ $\Rightarrow$ 微分式：$\nabla\times\vec{E}=-\frac{\partial\vec{B}}{\partial t}$

装

订

线

· 电势能 & 电势 无穷远处电势能为0 $U_\infty=0$

结合能 $U(r)=\frac{q_1q_2}{4\pi\varepsilon_0r}$ $U_p=W_{p\infty}=\int_p^\infty q\vec{E}\cdot d\vec{l}$

电势 $V=\frac{U}{q_0}$ $W_{AB}=-\Delta U$ $\Rightarrow\Delta V=V_b-V_a=-\int_a^b\vec{E}\cdot d\vec{l}$

$W_{AB}=\int_a^b\vec{F}\cdot d\vec{l}=q_0\int_a^b\vec{E}\cdot d\vec{l}=-\Delta U$

$\Rightarrow V_p=\int_p^\infty\vec{E}\cdot d\vec{l}$ $V_a-V_b=\frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r_a}-\frac{1}{r_b}\right)$.

$V(r)=\frac{q}{4\pi\varepsilon_0}\cdot\frac{1}{r}$

球壳 $r>R$ 时. $V(r)=\int_r^\infty\vec{E}\cdot d\vec{l}=\frac{q}{4\pi\varepsilon_0r}$

$r<R$ 时. $V(r)=\int_r^\infty\vec{E}\cdot d\vec{l}=\int_r^R\vec{E}\cdot d\vec{l}+\int_R^\infty\vec{E}\cdot d\vec{l}=\frac{q}{4\pi\varepsilon_0R}$

· 静电感应 导体

尖端放电： ps. 导体相连后 电势相同！电荷不一定！！

$\frac{Q_R}{4\pi\varepsilon_0R}=\frac{Q_r}{4\pi\varepsilon_0r}$ $\Rightarrow\frac{Q_R}{Q_r}=\frac{R}{r}$

$\sigma=\frac{Q}{4\pi r^2}$ $\frac{\sigma_R}{\sigma_r}=\frac{Q_R}{R^2}\cdot\frac{r^2}{Q_r}=\frac{r}{R}$.

$\Rightarrow r\downarrow\ \sigma_r\uparrow$ 面电荷密度↑.

$W=-\Delta U$

· $dW=-q_0dV=\vec{F}\cdot d\vec{l}=q_0E\cdot dl\cos\theta$

$\Rightarrow E\cos\theta=-\frac{dV}{dl}$ $\Rightarrow$ $\boxed{E_l=-\frac{dV}{dl}}$

$\Rightarrow\vec{E}=-\vec{\nabla}V$ $\vec{\nabla}V=\frac{\partial V}{\partial x}\hat{x}+\frac{\partial V}{\partial y}\hat{y}+\frac{\partial V}{\partial z}\hat{z}$

球坐标系：$\vec{\nabla}V=\frac{\partial V}{\partial r}\hat{r}+\frac{\partial V}{\partial\theta}\hat{\theta}\cdot\frac{1}{r}+\frac{1}{r\sin\theta}\cdot\frac{\partial V}{\partial\varphi}\hat{\varphi}$

· 电容器. $C=\frac{Q}{V}$

$C=\frac{\varepsilon_0A}{d}$ 电介质 k_e $C=\frac{k_e\varepsilon_0A}{d}$

$C=\frac{2\pi\varepsilon_0L}{\ln\frac{b}{a}}$ $C=\frac{2\pi\varepsilon_0k_eL}{\ln\frac{b}{a}}$

球壳 $C=4\pi\varepsilon_0\cdot\frac{ab}{b-a}=\frac{4\pi\varepsilon_0}{\frac{1}{a}-\frac{1}{b}}$ $C=k_e\frac{4\pi\varepsilon_0}{\frac{1}{a}-\frac{1}{b}}$

$\Rightarrow b\to\infty$ 时. $C=4\pi\varepsilon_0a$ 一个球也是电容器

串联：$C_串$：$\frac{1}{C_串}=\frac{1}{C_1}+\frac{1}{C_2}+\dots+\frac{1}{C_n}$ 并联 $C_并=C_1+C_2+\dots+C_n$

能量 $U=\frac{1}{2}CV^2=\frac{Q^2}{2C}$ $\Rightarrow dW=V(q)\cdot dq=\frac{q}{C}\cdot dq$ $\Rightarrow W=\frac{Q^2}{2C}$.

能量密度 $u=\frac{U}{V}=\frac{1}{2}\varepsilon_0E^2$ 普适

一半能量 R_0 半径 $\Rightarrow R_0=2R$

电介质：电场中的绝缘体 $C=k_eC_0$

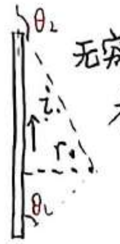
$\oint\vec{D}\cdot d\vec{A}=q_0$

$\Rightarrow4\pi r^2\cdot k_e\varepsilon_0E=q_0$ $\Rightarrow E=\frac{q_0}{4\pi k_er^2\varepsilon_0}$

· 磁场

$d\vec{F}_{12}=\frac{\mu_0}{4\pi}\cdot\frac{i_2 d\vec{s}_2\times(i_1 d\vec{s}_1\times\hat{r}_{12})}{r_{12}^2}$ $d\vec{F}_{21}=\frac{\mu_0}{4\pi}\times\frac{i_1 d\vec{s}_1\times(i_2 d\vec{s}_2\times\hat{r}_{21})}{r_{21}^2}$ 定义：$d\vec{F}_2=i_2 d\vec{s}_2\times\vec{B}_1$ $\vec{B}_1=\frac{\mu_0}{4\pi}\int_{L_1}\frac{i_1 d\vec{s}_1\times\hat{r}_{12}}{r^2}$

毕奥-萨阀尔定律：$d\vec{B}=\frac{\mu_0}{4\pi}\cdot\frac{i d\vec{s}\times\hat{r}}{r^2}$ $\vec{B}=\int_L\frac{\mu_0}{4\pi}\cdot\frac{i d\vec{s}\times\hat{r}}{r^2}$



无穷长：$B=\frac{\mu_0 i}{2\pi r_0}$

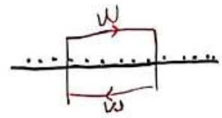
有限长：$\theta_1\to\theta_2$

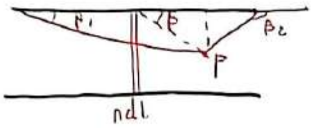
$B=\frac{\mu_0 i}{4\pi r_0}(\cos\theta_1-\cos\theta_2)$.

$B=\frac{\mu_0}{2}\cdot\frac{iR^2}{(R^2+r_0^2)^{3/2}}$. 中心处：$B=\frac{\mu_0 i}{2R}$

螺线管 Solenoid 单位长度 n 匝

Ampere环路定理解决：

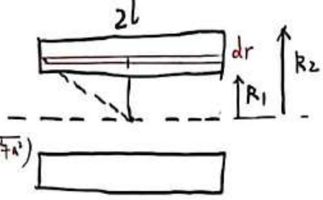




无限长 ⇒ $B=\mu_0 n i$. $\oint\vec{B}\cdot d\vec{l}=2Bw=\mu_0 i n w$

$B=\frac{1}{2}\mu_0 n i(\cos\beta_1-\cos\beta_2)$. $\Rightarrow B=\frac{1}{2}\mu_0 i n$ ⇒ 两个加起来 $\frac{1}{2}\mu_0 n i\times 2=\mu_0 n i$.

⇒ 推广 多层螺线管，考虑中心处的B.



$B=\frac{1}{2}\mu_0 n i(\cos\beta_1-\cos\beta_2)=\mu_0 n i\cos\beta_1$

$\cos\beta_1=\frac{l}{\sqrt{l^2+r^2}}$

$ni=\frac{N dr}{2l(R_2-R_1)}\cdot i$

$\Rightarrow dB=\mu_0\cdot\frac{Ni\cdot\frac{l}{\sqrt{l^2+r^2}}}{2l(R_2-R_1)}dr=\frac{\mu_0 N i}{2(R_2-R_1)}\cdot\frac{1}{\sqrt{l^2+r^2}}dr$

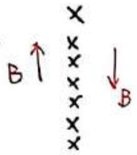
$\int\frac{1}{\sqrt{x^2+a^2}}dx=\ln(x+\sqrt{x^2+a^2})$

$\frac{N}{2l(R_2-R_1)}=J$

$\Rightarrow B=\mu_0 j l\int_{R_1}^{R_2}\frac{1}{\sqrt{l^2+r^2}}dr=\mu_0 j l\ln\frac{R_2+\sqrt{R_2^2+r^2}}{R_1+\sqrt{R_1^2+r^2}}$

· Magnetic Flux 磁通量 $\oint\vec{B}\cdot d\vec{A}=0$ $T\cdot m^2$ (Wb)

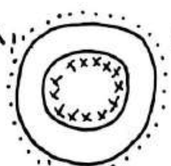
· Ampere's Loop Law：$\oint\vec{B}\cdot d\vec{l}=\mu_0\Sigma i$



$\oint\vec{B}\cdot d\vec{l}=Bw+0+Bw+0=2Bw=\mu_0 n w i$

$\Rightarrow B=\frac{1}{2}\mu_0 n i$

螺绕环：



$\oint\vec{B}\cdot d\vec{l}=B\cdot 2\pi r=\mu_0 n\cdot 2\pi r\, i$

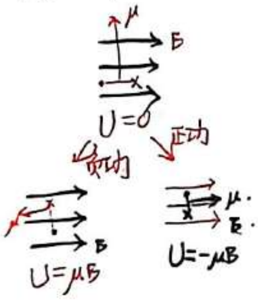
$\Rightarrow B=\mu_0 n i$.

磁偶极矩 Vs 电偶极矩.

$\vec{\tau}=\vec{\mu}\times\vec{B}$ $\vec{\tau}=\vec{p}\times\vec{E}$

$U=-\int\vec{\tau}\cdot d\vec{\theta}$

势能 $\vec{\mu}\cdot\vec{B}$ $\vec{p}\cdot\vec{E}$



$U=0$

负功 正功

$U=\mu B$ $U=-\mu B$

· 洛仑兹力 $\vec{F}=q\vec{v}\times\vec{B}$ $F=qvB\sin\theta$

· 霍尔效应：$qvB=qE\Rightarrow E=vB$.

$V_{霍尔}=E\cdot b=vB\cdot b=\frac{j}{nq}B\cdot b=\frac{1}{nq}\cdot\frac{iB}{d}$.

$R_H=\frac{B}{nqd}$.

$R_H=\frac{V_{霍尔}}{i}$

· 互感 (mutual inductance).

$M_{12}=\frac{\psi_{12}}{i_2}=\frac{N_2\Phi_{12}}{i_2}$ $\Rightarrow\varepsilon_2=-\frac{d\psi_{12}}{dt}=-M_{12}\frac{di_2}{dt}$

· 自感 $\psi=NBA=Li$. $L=\frac{\Phi_B}{i}$

$\varepsilon_L=-\frac{d\psi}{dt}=-L\frac{di}{dt}$

两个线圈 N_1, L_1 与 N_2, L_2.

① 无通量关系时. $M=\sqrt{L_1L_2}$

② 連在一起时：同向 $L=L_1+L_2+2\sqrt{L_1L_2}$

异向 $L=L_1+L_2-2\sqrt{L_1L_2}$

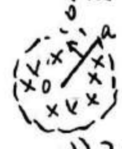
$L=k_m L_0$ → 磁导率.

· 电磁感应定律：$\Phi_B=\iint\vec{B}\cdot d\vec{A}$

$\Rightarrow\varepsilon=-\frac{d\Phi_B}{dt}$ → 楞次定律

① 动生电动势 $\varepsilon=\int_-^+(\vec{v}\times\vec{B})\cdot d\vec{l}$

ε：非静电力将电荷正电荷从负极到正极做功.



$d\varepsilon=(\vec{v}\times\vec{B})\cdot d\vec{r}=-Bvdr$.

$\varepsilon=-\int_0^R B\omega r dr=-\frac{1}{2}B\omega R^2$.

② 感生电动势；(涡旋电场) $\vec{E}'$ $\oint\vec{E}\cdot d\vec{l}=-\iint\frac{\partial\vec{B}}{\partial t}\cdot d\vec{A}$

$\varepsilon=\oint\vec{E}\cdot d\vec{l}$

总体有：$\varepsilon=\oint(\vec{E}'+\vec{v}\times\vec{B})\cdot d\vec{l}$

感生 动生.

· 原子核磁性：→ 轨道磁矩 原子 $\mu_{PL}\neq 0$ 中子为0

$\mu_l=\frac{e}{2m}l$ $\therefore\vec{\mu}_l=-\frac{e}{2m}\vec{l}$

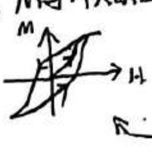
$l=mvr$ 角动量 自旋磁矩

$\vec{\mu}_{PS}\neq 0$ $\vec{\mu}_{ns}\neq 0$

铁磁材料 温度↑会向顺磁性转变 $\chi_m=\frac{M}{H}=\frac{m}{T-\theta}$ Curie-Weiss Law

$\theta>0$ 是铁磁材料特征参数

磁滞现象 与磁畴的转向与排布有关，M与H关系曲线有滞后



电流

电介质 $\begin{cases} K_e \text{ 介电常数} \\ \chi_e \text{ 极化率} \\ \vec{P} \text{ 极化强度矢量} \\ \vec{D} \text{ 电位移矢量} \end{cases}$

$\Downarrow$

$C = K_e C_0$

$K_e = 1 + \chi_e$

$P = \chi_e \varepsilon_0 E$

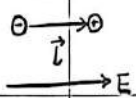
$\oint (\varepsilon_0 \vec{E} + \vec{P}) \cdot d\vec{A} = \sum q_0$

$\vec{D} = \varepsilon_0 \vec{E} + \vec{P} = \varepsilon_0 \vec{E} + \chi_e \varepsilon_0 \vec{E}$
$= (1+\chi_e)\varepsilon_0 \vec{E}$
$= K_e \varepsilon_0 \vec{E}$

PS. $\nabla \cdot \vec{D} = \rho_f$
$\nabla \cdot \vec{P} = -\rho_b$

取向沿 $\vec{E}$

$\vec{P} = \dfrac{\sum p_m}{\Delta V}$ → p_m：感生电偶极矩



$\vec{P} = \dfrac{\sum p_m}{\Delta V} = nq\vec{l}$ （n：单位体积分子数）

$\therefore dN = n\,dV = n\,l\,dA\cos\theta$

$dq = q\,dN = n\,l\,q\,dA\cos\theta$
$= P\,dA\cos\theta = \vec{P} \cdot d\vec{A}$

$\Rightarrow \oint \vec{P} \cdot d\vec{A} = \sum_{out} q' = -\sum_{in} q'$

$\sigma' = \dfrac{dq}{dA} = P\cos\theta = \vec{P} \cdot \vec{n} = P_n$

$E = E_0 + E'$ $\quad P = \chi_e \varepsilon_0 E$

Gauss' Law: $\oint \vec{E} \cdot d\vec{A} = \dfrac{\sum q}{\varepsilon_0}$

$\Rightarrow \varepsilon_0 \oint \vec{E} \cdot d\vec{A} = \sum q$ $\quad \sum_{in} q'$

$\Downarrow$

$\varepsilon_0 \oint (\vec{E}_0 + \vec{E}') d\vec{A} = \sum q_0 + \sum_{in} q'$

$\Downarrow$

$\Rightarrow \varepsilon_0 \oint \vec{E} \cdot d\vec{A} + \oint \vec{P} \cdot d\vec{A} = \sum q_0$ $\quad -\sum_{in} q' = -\oint \vec{P} \cdot d\vec{A}$

$\Rightarrow \oint (\varepsilon_0 \vec{E} + \vec{P}) \cdot d\vec{A} = \sum q_0$

$\Rightarrow \vec{D} = \varepsilon_0 \vec{E} + \vec{P}$

$\oint \vec{D} \cdot d\vec{A} = \sum q_0$ 高斯定理推广

$L = K_m L_0$ → 自感

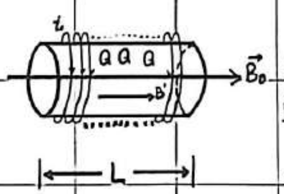
磁介质 $\begin{cases} K_m \text{ 磁导率} \\ \chi_m \text{ 磁化率} \\ M \text{ 磁化强度矢量} \\ H \text{ 磁场强度} \end{cases}$

$\Downarrow$

顺磁 同向↑
抗磁 异向↓
铁磁 同向↑↑

$\vec{M} = \chi_m \vec{H}$

$\vec{B} = K_m \mu_0 \vec{H}$



环状分子电流（i_m），组成束缚电流
单位长度密度为 j'

$\Delta V = AL$ $\quad \sum \mu_m = \sum i_m \Delta A = j' L A$ $\Rightarrow j' = \dfrac{\sum \mu_m}{\Delta V}$

$\vec{M} \times \vec{n} = \vec{j}'$

$\Uparrow$

单位体积磁偶极矩 $\vec{M} = \dfrac{\sum \vec{\mu}_m}{\Delta V}$ $\quad \oint \vec{M} \cdot d\vec{l} = \sum i'$ 束缚电流

$\vec{B} = \vec{B}_0 + \vec{B}_m$ $\quad \oint_L \vec{M} \cdot d\vec{l} = \sum i'$

$\oint_L \vec{B} \cdot d\vec{l} = \mu_0 \sum i = \mu_0 \sum (i_0 + i') = \mu_0 \sum i_0 + \mu_0 \sum i' = \mu_0 \sum i_0 + \mu_0 \oint_L \vec{M} \cdot d\vec{l}$

$\Rightarrow \oint_L \left(\dfrac{\vec{B}}{\mu_0} - \vec{M}\right) \cdot d\vec{l} = \sum i_0$ $\quad \oint_L \vec{H} \cdot d\vec{l} = \sum i_0$ $\quad$ 真空中 $\vec{M} = 0$ $\vec{H} = \dfrac{\vec{B}}{\mu_0}$

$\Rightarrow \vec{B} = \mu_0 \vec{H}$

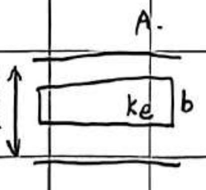
$\vec{H} = \dfrac{\vec{B}}{\mu_0} - \vec{M}$

$\begin{cases} \vec{M} = \chi_m \vec{H} \\ \vec{B} = K_m \mu_0 \vec{H} \end{cases}$

$H \cdot \Delta l = N i_0 \Rightarrow H = n i_0$ （n：单位长度数）

对于圆柱体铁磁介质：$B_0 = \mu_0 n i_0$ $\quad B = K_m \mu_0 H = K_m \mu_0 n i_0 = K_m B_0 \Rightarrow \dfrac{B}{B_0} = K_m$

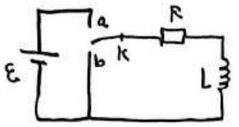
补：



$C_1 = \dfrac{\varepsilon_0 A}{d-b}$ $\quad C_2 = \dfrac{K_e \varepsilon_0 A}{b}$

$\dfrac{1}{C} = \dfrac{1}{C_1} + \dfrac{1}{C_2}$

RL电路、 ①充电 a



$iR + L\frac{di}{dt} = \varepsilon$

$\frac{di}{dt} = \frac{1}{L}(\varepsilon - iR) = -\frac{R}{L}(i - \frac{\varepsilon}{R})$. $\Rightarrow i - \frac{\varepsilon}{R} = C'e^{-\frac{R}{L}t}$ $C' = -\frac{\varepsilon}{R}$

$\Rightarrow i = \frac{\varepsilon}{R}(1 - e^{-Rt/L})$. L上电压：$V_L = L\frac{di}{dt} = \varepsilon e^{-Rt/L}$.

②放电 b. $iR + L\frac{di}{dt} = 0 \Rightarrow i = \frac{\varepsilon}{R}e^{-Rt/L}$ $V_L = L\frac{di}{dt} = -\varepsilon e^{-Rt/L}$

能量：eg. Capacitor：$U_e = \frac{Q^2}{2C} = \frac{1}{2}CV^2$ $u_E = \frac{1}{2}\varepsilon_0 E^2$

L中：$dW = -\varepsilon_L dq = -\varepsilon_L i dt \Rightarrow dW = Li\,di \Rightarrow W = \frac{1}{2}L i_{max}^2 = \frac{1}{2}LI^2$

能量密度：$u_B = \frac{U}{V} = \frac{\frac{1}{2}LI^2}{lA} = \frac{\frac{1}{2}\mu_0 n^2 lAI^2}{lA} = \frac{1}{2}\mu_0 n^2 I^2 = \frac{B^2}{2\mu_0}$ $u_e = \frac{1}{2}\varepsilon_0 E^2$.

Vacuum：$u_B = \frac{B^2}{2\mu_0} = \frac{1}{2}\vec{B}\cdot\vec{H}$ $u_E = \frac{1}{2}\varepsilon_0 E^2 = \frac{1}{2}\vec{D}\cdot\vec{E}$

$\Rightarrow$ 两个线圈 $\begin{cases} N_1 L_1 \\ N_2 L_2 \end{cases}$. 有互感 $\begin{cases} M_{12} \\ M_{21} \end{cases}$

互感能量

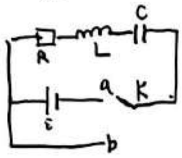
$W = W_1 + W_2 = -\int_0^\infty \varepsilon_{21} i_1 dt - \int_0^\infty \varepsilon_{12} i_2 dt$

$= MI_1I_2 = \frac{1}{2}M_{12}I_1I_2 + \frac{1}{2}M_{21}I_1I_2$

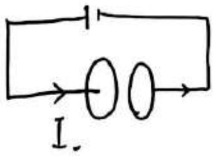
$U = \frac{1}{2}L_1I_1^2 + \frac{1}{2}L_2I_2^2 + MI_1I_2$

电磁振荡：$U = U_B + U_E = \frac{1}{2}Li^2 + \frac{1}{2}\frac{q^2}{C} \Rightarrow \frac{dU}{dt} = Li\frac{di}{dt} + \frac{q}{C}\frac{dq}{dt} = Li\frac{d^2q}{dt^2} + \frac{q}{C}i = 0 \Rightarrow \frac{d^2q}{dt^2} + \frac{1}{LC}q = 0. \Rightarrow \omega = \sqrt{\frac{1}{LC}}$.

$L\frac{di}{dt} + iR + \frac{q}{C} = \begin{cases} \varepsilon \\ 0 \end{cases}$



阻尼 $\begin{cases} \text{过} \\ \text{临界} \\ \text{阻尼振荡} \end{cases}$



位移电流：$\oint \vec{H}\cdot d\vec{l} = i_0 + i_D$.

平行板电容器.

$E = \frac{\sigma}{\varepsilon_0} = \frac{q}{\varepsilon_0 A}$.

$\oiint \vec{D}\cdot d\vec{A} = q_0$ $\oiint \vec{j}_0\cdot d\vec{A} = -\frac{dq_0}{dt} = -\oiint \frac{\partial\vec{D}}{\partial t}\cdot d\vec{A} \Rightarrow \oiint(\vec{j}_0 + \frac{\partial\vec{D}}{\partial t})d\vec{A} = 0$. $q = \varepsilon_0 AE$

新安培环路定理：$\oint \vec{H}\cdot d\vec{l} = i_0 + i_D = \iint(\vec{j}_0 + \frac{\partial\vec{D}}{\partial t})\cdot d\vec{A}$ $i_0 = \frac{dq}{dt} = \varepsilon_0\frac{d\Phi_E}{dt} = \frac{d\Phi_D}{dt} = i_D$ A.

麦克思韦方程组：

$$\begin{cases} \oiint \vec{D}\cdot d\vec{A} = q_0 \\ \oiint \vec{B}\cdot d\vec{A} = 0 \\ \oint \vec{E}\cdot d\vec{l} = -\iint \frac{\partial\vec{B}}{\partial t}d\vec{A} \\ \oint \vec{H}\cdot d\vec{l} = i_0 + \iint \frac{\partial\vec{D}}{\partial t}d\vec{A} \end{cases} \longrightarrow \begin{cases} \nabla\cdot\vec{D} = \rho_{e0}. \\ \nabla\cdot\vec{B} = 0 \\ \nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} \\ \nabla\times\vec{H} = \vec{j}_0 + \frac{\partial\vec{D}}{\partial t}. \end{cases}$$

$\rho_{e0} = 0$. $j_0 = 0$.

真空中：
$$\begin{cases} \nabla\cdot\vec{E} = 0 \\ \nabla\cdot\vec{B} = 0 \rightarrow \nabla\cdot\vec{H} = 0 \\ \nabla\times\vec{E} = -k_m\mu_0\frac{\partial\vec{H}}{\partial t} \\ \nabla\times\vec{H} = k_e\varepsilon_0\frac{\partial\vec{E}}{\partial t}. \end{cases}$$

$$\Rightarrow \begin{cases} \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} = 0 \\ \frac{\partial H_x}{\partial x} + \frac{\partial H_y}{\partial y} + \frac{\partial H_z}{\partial z} = 0 \\ \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix} = -k_m\mu_0\left(\frac{\partial H_x}{\partial t}\vec{i} + \frac{\partial H_y}{\partial t}\vec{j} + \frac{\partial H_z}{\partial t}\vec{k}\right). \\ \cdots\cdots \end{cases}$$

用略.

$$\Rightarrow \begin{cases} \frac{\partial^2 E_x}{\partial z^2} - k_e\varepsilon_0 k_m\mu_0\frac{\partial^2 E_x}{\partial t^2} = 0 \\ \frac{\partial^2 H_y}{\partial z^2} - k_e\varepsilon_0 k_m\mu_0\frac{\partial^2 H_y}{\partial t^2} = 0 \end{cases} \Rightarrow v = \frac{1}{\sqrt{k_e\varepsilon_0 k_m\mu_0}}$$

$\sqrt{k_e\varepsilon_0}E_0 = \sqrt{k_m\mu_0}H_0$.

真空中：$k_m = k_e = 1 \Rightarrow \sqrt{\varepsilon_0}E_0 = \sqrt{\mu_0}H_0$.

$\Rightarrow B_0 = \frac{E_0}{c}$. $c = \frac{1}{\sqrt{\varepsilon_0\mu_0}}$.

实验名称：________ 姓名：________ 学号：________

坡印廷矢量 $\vec{S}=\vec{E}\times\vec{H}$ (单位时间内通过单位面积的电磁能量).

电磁波能量：$U=\iiint\left(\frac{1}{2}\varepsilon_0E^2+\frac{B^2}{2\mu_0}\right)dV = \iiint\left(\frac{1}{2}\vec{D}\cdot\vec{E}+\frac{1}{2}\vec{B}\cdot\vec{H}\right)dV \Rightarrow \frac{dU}{dt}=-\oint\vec{S}\cdot d\vec{A}-Q+P.$

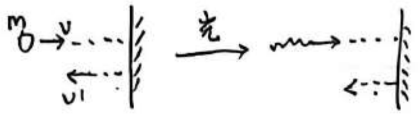
电磁波中 $u_E=u_B$ [$u_E=\frac{1}{2}\varepsilon_0E^2$; $u_B=\frac{B^2}{2\mu_0}$ $\sqrt{K_m\mu_0}H=\sqrt{K_e\varepsilon_0}E$. 真空中 $\Rightarrow \sqrt{\mu_0}H=\sqrt{\varepsilon_0}E$.

$u_E=\frac{1}{2}\varepsilon_0E^2=\frac{1}{2}\varepsilon_0c^2B^2=\frac{1}{2}\varepsilon_0\frac{1}{\varepsilon_0\mu_0}B^2=\frac{B^2}{2\mu_0}=u_B$]

$\therefore u=u_B+u_E=\varepsilon_0E^2$

平均能量密度 $\langle u\rangle=\varepsilon_0\langle E^2\rangle=\varepsilon_0E_{max}^2\langle\sin^2(\omega t-kx)\rangle=\frac{\varepsilon_0E_{max}^2}{2}$ $\quad\langle u\rangle=\varepsilon_0E_{rms}^2=\varepsilon_0\left(\frac{E_{max}}{\sqrt{2}}\right)^2.$

波的强度：$I=c\langle u\rangle=c\cdot\frac{\varepsilon_0E_{max}^2}{2}$ (单位时间内通过单位面积的电磁能).



$\Delta\vec{F}\cdot c\Delta t=(\vec{S}_{in}-\vec{S}_{ref})\Delta A\cdot\Delta t.$

$\therefore \Delta\vec{F}=\frac{1}{c}(\vec{S}_{in}-\vec{S}_{ref})\Delta A$

光压：$P=\frac{|\Delta\vec{F}|}{\Delta A}=\frac{1}{c}(|\vec{S}_{in}|+|\vec{S}_{ref}|)$

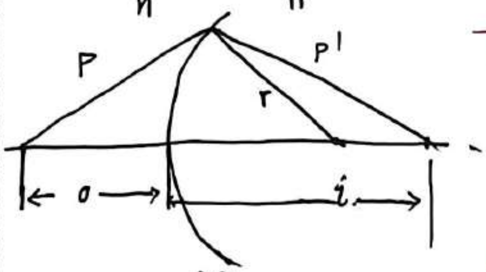
$\Downarrow$ 全反射

$P=\frac{2S}{c}=\frac{2}{c}EH$

动量：$\Delta\vec{G}_p=\Delta\vec{F}\cdot\Delta t=\frac{1}{c}(\vec{S}_{in}-\vec{S}_{ref})\Delta A\cdot\Delta t.$

动量密度：$\frac{\Delta G_p}{\Delta V}=\frac{\frac{1}{c}(\vec{S}_{in}-\vec{S}_{ref})\Delta A\Delta t}{\Delta A\cdot c\cdot\Delta t}=\frac{1}{c^2}(\vec{S}_{in}-\vec{S}_{ref})=\Delta\vec{g}$ $\begin{cases}\vec{g}_{in}=\frac{\vec{S}_{in}}{c^2}\\ \vec{g}_{out}=\frac{\vec{S}_{ref}}{c^2}\end{cases}$

·球面镜成像：



$\frac{n}{o}+\frac{n'}{i}=\frac{n'-n}{r}$

$i\to\infty$时，第一焦点：$o=f=\frac{nr}{n'-n}$

$o\to\infty$时，第二焦点：$i=f'=\frac{n'r}{n'-n}$

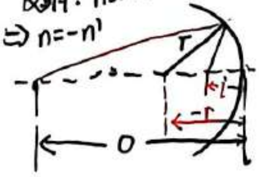
$\frac{f}{f'}=\frac{n}{n'}$ $\quad \frac{f}{o}+\frac{f'}{i}=1.$

符号：o >0 实物，<0 虚物
i >0 实像，<0 虚像
r >0 凸，<0 凹

·球面反射成像.$(n=-n')$

反射：$n\sin\theta=n'\sin\theta'$

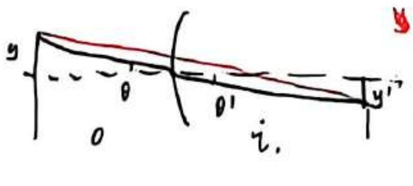
$\Rightarrow n=-n'$



由 $\frac{n}{o}+\frac{n'}{i}=\frac{n'-n}{r}$; $\frac{f}{o}+\frac{f'}{i}=1.$

$\Rightarrow \frac{n}{o}+\frac{-n}{(-i)}=\frac{-2n}{r} \Rightarrow \frac{1}{o}+\frac{1}{i}=-\frac{2}{r}.$

·横向放大率：$m=\frac{y'}{y}=-\frac{i\theta'}{o\theta}=-\frac{ni}{n'o}$ $\quad\Rightarrow$ 对反射成像：$m=\frac{i}{o}.$

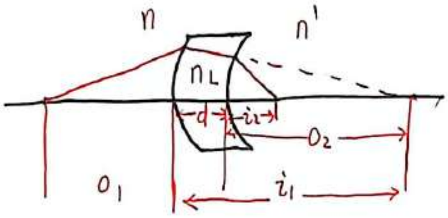


$n\theta\approx n'\theta'.$

$y=o\theta$

$y'=i\theta'.$ $\Rightarrow \frac{y'}{y}=-\frac{i\theta'}{o\theta}=-\frac{i}{o}\frac{n}{n'}.$

4、磨镜者公式（薄透镜）.



$o_2 \fallingdotseq -(i_1 - d)$ $\quad d \approx 0$

$\Rightarrow o_2 \fallingdotseq -i_1$

①、$n \to n_L$.

$$\begin{cases} \dfrac{n}{o_1} + \dfrac{n_L}{i_1} = \dfrac{n_L - n}{r_1} \\ \dfrac{f_1}{o_1} + \dfrac{f_1'}{i_1} = 1. \end{cases}$$

$\Downarrow$

$$\begin{cases} f_1 = \dfrac{n r_1}{n_L - n} \\ f_1' = \dfrac{n_L r_1}{n_L - n} \end{cases}$$

②、$n_L \to n'$.

$$\begin{cases} \dfrac{n_L}{o_2} + \dfrac{n'}{i_2} = \dfrac{n' - n_L}{r_2} \\ \dfrac{f_2}{o_2} + \dfrac{f_2'}{i_2} = 1. \end{cases}$$

$\Downarrow$

$$\begin{cases} f_2 = \dfrac{n_L r_2}{n' - n_L}. \\ f_2' = \dfrac{n' r_2}{n' - n_L}. \end{cases}$$

$$\Rightarrow \begin{cases} \dfrac{f_1}{o_1} + \dfrac{f_1'}{i_1} = 1 \longrightarrow \dfrac{f_1 f_2}{o_1} + \dfrac{f_1' f_2}{i} = f_2 \\ \dfrac{f_2}{-i_1} + \dfrac{f_2'}{i_2} = 1. \longrightarrow \dfrac{f_1' f_2}{-i_1} + \dfrac{f_1' f_2'}{i_2} = f_1' \end{cases}$$

相加 $\Rightarrow \dfrac{f_1 f_2}{o_1} + \dfrac{f_1' f_2'}{i_2} = f_1' + f_2$

$\Rightarrow$ 整个过程：$\dfrac{f_1 f_2}{o} + \dfrac{f_1' f_2'}{i} = f_1' + f_2 \quad \Rightarrow \begin{cases} f = \dfrac{f_1 f_2}{f_1' + f_2} \\ f' = \dfrac{f_1' f_2'}{f_1' + f_2}. \end{cases}$

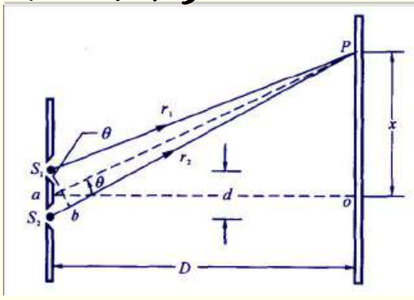
$$f = \frac{\dfrac{n n_L r_1 r_2}{(n_L - n)(n' - n_L)}}{\dfrac{n_L r_1}{n_L - n} + \dfrac{n_L r_2}{n' - n_L}} = \frac{n n_L r_1 r_2}{(n' - n_L) n_L r_1 + (n_L - n) n_L r_2} = \frac{n}{\dfrac{(n' - n_L)}{r_2} + \dfrac{(n_L - n)}{r_1}}$$

$$f' = \frac{n'}{\dfrac{n_L - n}{r_1} + \dfrac{n' - n_L}{r_2}} \quad \Rightarrow \quad \frac{f'}{f} = \frac{n'}{n}$$

$n = n' = 1$ 时

$$\Rightarrow f = f' = \frac{1}{(n_L - 1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)}$$

9. 光的干涉



$\delta = r_2 - r_1 \approx d\sin\theta$

极大：$\delta = d\sin\theta = k\lambda$

极小：$\delta = d\sin\theta = (k-\frac{1}{2})\lambda$

$k\in Z$.

$\Delta\varphi = 2\pi\cdot\frac{\delta}{\lambda} \approx \frac{2\pi}{\lambda}\cdot d\sin\theta$

观察屏上位置：$\begin{cases} x = \frac{D}{d}k\lambda \\ x = \frac{D}{d}(k-\frac{1}{2})\lambda \end{cases}$ $k\in Z$

相干光波：频率相同、振动方向相同、相位差恒定

相邻明/暗纹：$\Delta x = \frac{D}{d}\lambda$

$\begin{cases} E_1 = E_{10}\cos(\omega t+\varphi_1) \\ E_2 = E_{20}\cos(\omega t+\varphi_2) \end{cases}$ $\Rightarrow$ 合成 $\vec{E} = \vec{E}_1 + \vec{E}_2 = E_0\cos(\omega t+\varphi)$.

$E_0\cos\varphi = E_{10}\cos\varphi_1 + E_{20}\cos\varphi_2$

$E_0\sin\varphi = E_{10}\sin\varphi_1 + E_{20}\sin\varphi_2$

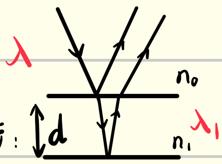
$\varphi = \arctan\frac{E_{10}\sin\varphi_1 + E_{20}\sin\varphi_2}{E_{10}\cos\varphi_1 + E_{20}\cos\varphi_2}$

$\Downarrow$

$E_0^2 = E_{10}^2 + E_{20}^2 + 2E_{10}E_{20}\cos(\varphi_2-\varphi_1)$

光强：$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos(\varphi_2-\varphi_1)$ 中央附近 $I_1\approx I_2 = I_0$ $I = 2I_0[1+\cos(\varphi_2-\varphi_1)] = 4I_0\cos^2(\frac{\varphi_2-\varphi_1}{2})$

·薄膜干涉：



$n_0 > n_1$ 时，无损失

$n_0 < n_1$ 时，反射后有半波损失

① $\lambda_1 = \frac{v_1}{f} = \frac{c}{n_1 f} = \frac{\lambda}{n_1}$

$\frac{2d}{\lambda_1} = \frac{2d\cdot n_1}{\lambda}$ → 再判断是否半波损失与 $\begin{cases} = k \\ = k+\frac{1}{2} \end{cases}$

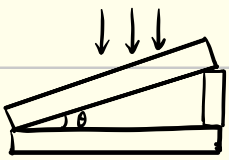
② 光程：nr 光程差 $\delta = n_2 r_2 - n_1 r_1$

$\because \Delta\varphi = \frac{2\pi}{\lambda}\cdot\delta = \frac{2\pi}{\lambda}(n_1\cdot 2d - 0)$ $\Rightarrow \frac{\Delta\varphi}{2\pi} = \frac{n_1\cdot 2d}{\lambda}$

相位差取决于光程差

而非几何路程差.

劈尖干涉：

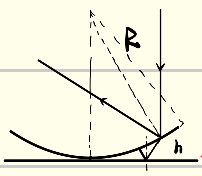


h 时，$\delta = 2nh = k\lambda$.

$h+\Delta h$ 时，$\delta = 2n(h+\Delta h) = (k+1)\lambda$ 下一级条纹

空气中 $n=1$.

$\therefore 2n\Delta h = \lambda$ $\Delta h = \frac{\lambda}{2n}$ $\Rightarrow \tan\theta = \frac{\Delta h}{\Delta x} \approx \theta$ $\therefore \theta \approx \frac{\lambda}{2n\Delta x} = \frac{\lambda}{2\Delta x}$



牛顿环：

有半波损失 $2nh + \frac{\lambda}{2} = \begin{cases} k\lambda, & 明纹 \\ (k+\frac{1}{2})\lambda, & 暗纹 \end{cases}$ $k\in Z$.

$n=1$; 明半径 $r = \sqrt{(k-\frac{1}{2})R\lambda}$

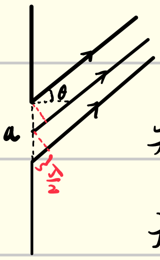
$k\in Z$.

暗半径：$r = \sqrt{k\lambda R}$.

5 衍射与光栅

衍射：遇障碍物偏离直线传播

Fraunhofer 单缝衍射



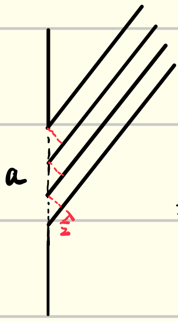
偶数个半波带

无数对这样的两条光线均干涉

相消 $\therefore \delta = a\sin\theta = 2k\cdot\frac{\lambda}{2}$

$\Rightarrow a\sin\theta = k\lambda$

暗



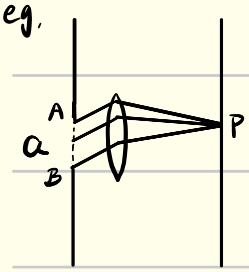
奇数个半波带，上部分两两成对

相消，下方始终还有一个半波带，⇒亮纹.

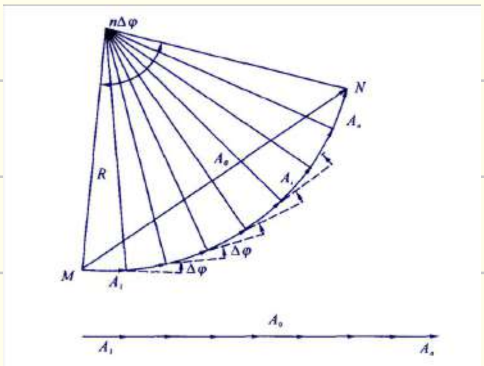
$\delta = a\sin\theta = (2k+1)\cdot\frac{\lambda}{2}$ $\Rightarrow a\sin\theta = (k+\frac{1}{2})\lambda$

· 矢量图解：由于在 a 处光是连续的，因此无数条光到 P 的

eg.



相位也是连续的从 A 到 B



$\overset{\frown}{MN} = R(n\Delta\varphi) = nA_1$

合振幅 $A_\theta = 2R\sin\frac{n\Delta\varphi}{2} = 2\frac{nA_1}{n\Delta\varphi}\sin\frac{n\Delta\varphi}{2}$

$n\Delta\varphi = \frac{2\pi}{\lambda}a\sin\theta$ (总光程差).

$\therefore A_\theta = nA_1\frac{\sin(\frac{\pi a}{\lambda}\sin\theta)}{\frac{\pi}{\lambda}a\sin\theta} = nA_1\frac{\sin u}{u}$

展成直线为 A_0
中央明纹中心
处的振幅
$\theta = 0$.

光强与振幅平方成正比！ $\Rightarrow A_\theta = A_0\frac{\sin u}{u}$.

光强：$\frac{I}{I_0} = \frac{A_\theta^2}{A_0^2} = \frac{\sin^2 u}{u^2}$ $\therefore I = I_0\frac{\sin^2 u}{u^2}$ $(u = \frac{\pi a}{\lambda}\sin\theta)$.

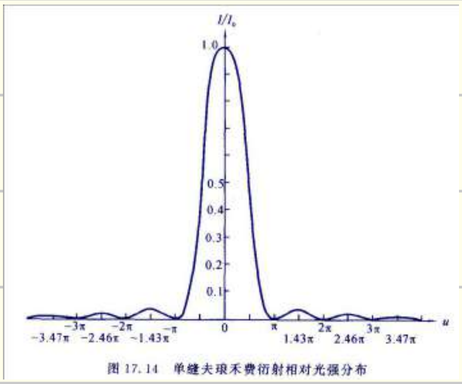


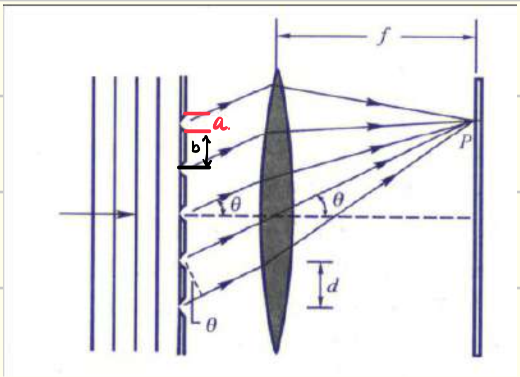
图 17.14 单缝夫琅禾费衍射相对光强分布

$u = \frac{n\Delta\varphi}{2} = \frac{\frac{2\pi}{\lambda}\cdot a\sin\theta}{2}$

当 $\frac{\pi a}{\lambda}\sin\theta = k\pi$，$k\in\mathbb{Z}$ 时，I 最小.

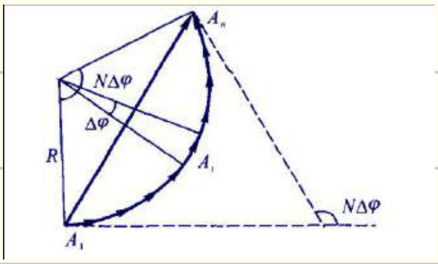
当 $\frac{\pi a}{\lambda}\sin\theta \to 0$ 时，$I = I_0$ 最大. $(\theta = 0)$

· 光栅衍射



相邻两缝

$d=a+b$ ，共N条缝，相邻两缝相位差：$\Delta\varphi=\frac{2\pi}{\lambda}d\sin\theta$



总相位差：$N\Delta\varphi$

$\therefore A=2R\sin\frac{N\Delta\varphi}{2}$

$A_i=2R\sin\frac{\Delta\varphi}{2}$ $\quad A_i=A_0\frac{\sin u}{u}$ $\quad u=\frac{\pi a\sin\theta}{\lambda}$

$$\Rightarrow A=\frac{A_i\sin\frac{N\Delta\varphi}{2}}{\sin\frac{\Delta\varphi}{2}}=A_0\frac{\sin u}{u}\cdot\frac{\sin\frac{N\Delta\varphi}{2}}{\sin\frac{\Delta\varphi}{2}}$$

$$\Rightarrow I=I_0\left(\frac{\sin u}{u}\right)^2\cdot\left(\frac{\sin\frac{N\Delta\varphi}{2}}{\sin\frac{\Delta\varphi}{2}}\right)^2$$

$u=\frac{\pi a\sin\theta}{\lambda}$ $\quad \frac{\Delta\varphi}{2}=\frac{\pi d\sin\theta}{\lambda}$

单缝 $\qquad$ 缝之间

光栅的分辨本领：把波长相近的两条谱线分辨清楚

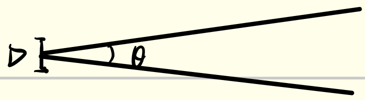
$R=\frac{\lambda}{\Delta\lambda}=kN$ （衍射级次 k）

λ → 平均波长

$\Delta\lambda$ → 波长差

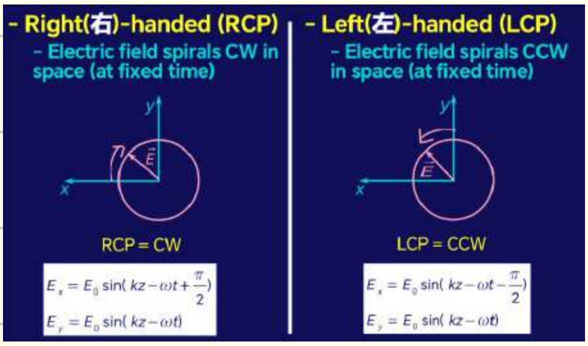
$d\sin\theta=k(\lambda+\Delta\lambda)$

$Nd\sin\theta=k'\lambda=(kN+1)\lambda$



圆孔衍射——最小分辨角 $\theta_{min}=1.22\frac{\lambda}{D}$

分辨本领：$R=\frac{1}{\theta_{min}}$



9. 偏振

· 线偏振光：传播过程中，空间各点光矢量都沿同一个固定方向振动。

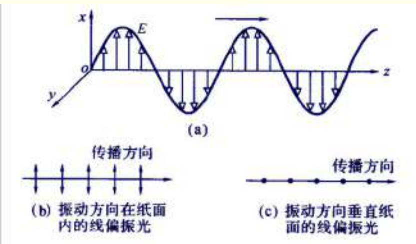
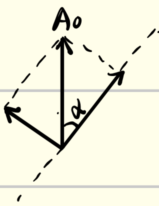


图 18.2 线偏振光

自然光在各个方向高速随机变化



$A=A_0\cos\alpha$. $\quad\therefore A^2=A_0^2\cos^2\alpha$

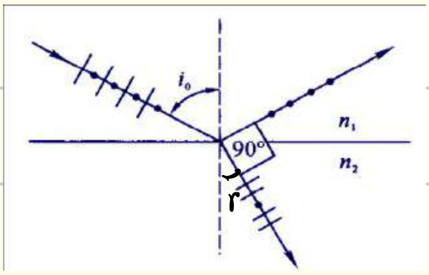
$\frac{I}{I_0}=\frac{A^2}{A_0^2}=\cos^2\alpha$

$\Rightarrow I=I_0\cos^2\alpha$ 马吕斯定律

自然光通过第一个偏振片后 光强为原来的一半!

布儒斯特定律：入射角某一i_0时，反射光成为线偏振光，振动方向垂直入射面，

折射光有全部平行振动成分，最少量的垂直振动成分。此时反射光与折射光垂直。



$$n_1\sin i_0 = n_2\sin(\underbrace{90°-i_0}_{r}) = n_2\sin r \Rightarrow \frac{n_2}{n_1}=\tan i_0$$

$\tan i_0 = n_{21}$ 媒质2对媒质1的相对折射率

i_0：布儒斯特角，起偏振角

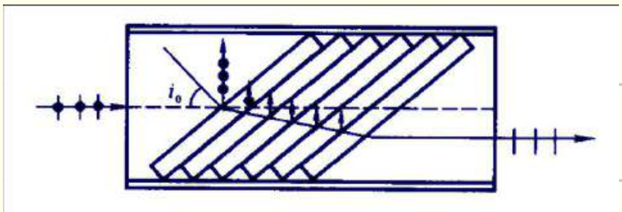


图 18.10 用玻璃片堆制成的偏振器

•光的双折射

寻常光线：o光，服从通常的折射定律；非常光线，e光 传播方向不在入射面内

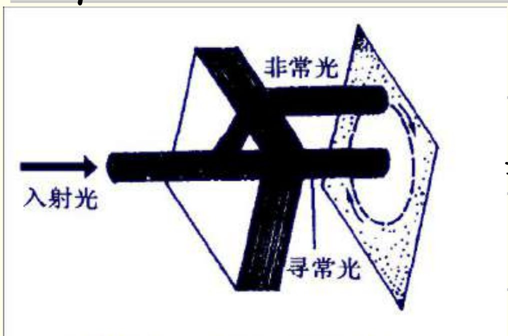


图 18.14 寻常光和非常光

都是线偏振光；振动方向几乎互相垂直

⇒垂直入射后，转动晶体，e光转动

光束沿某方向传播时不发生双折射，这一方向称为晶体的光轴

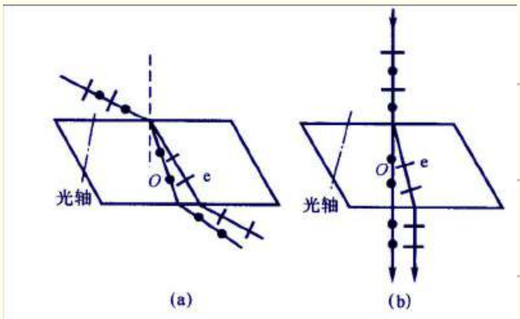


图 18.16 主平面(在纸面上)

原因：晶体的规则结构（各向异性）

e光不遵从折射定律，v也随方向而异

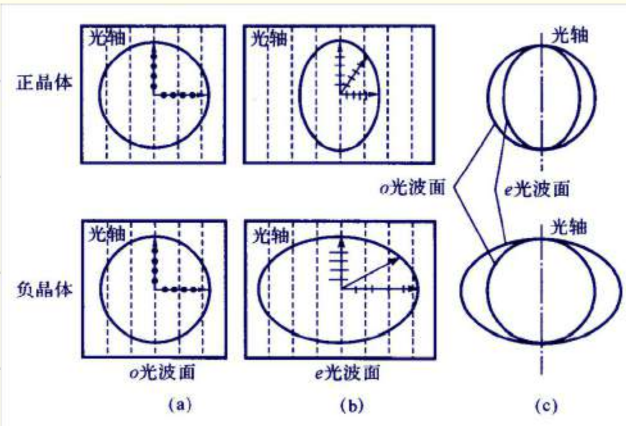


图 18.17 e 光和 o 光的波面

$n_e>n_o$

$n_e<n_o$

e光的主折射率：$n_e=\frac{c}{v_e}$（e沿垂直光轴方向的速度 v_e）. n_o, n_e 为晶体主折射率.

黑体辐射

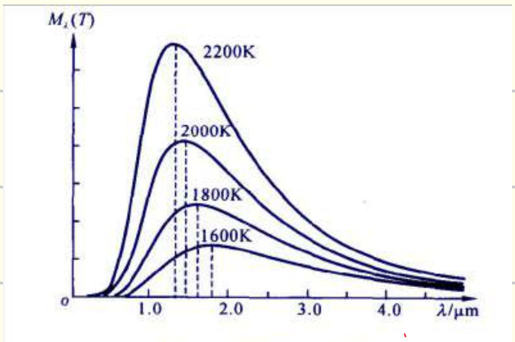
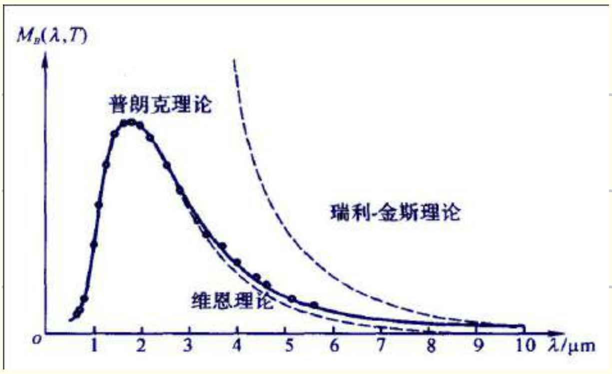


图 20.4 $M_\lambda(T)$—λ 曲线

辐射出射度 $M_B(T)=\int_0^\infty M_{B\lambda}(T)\,d\lambda=\sigma T^4$ 斯特藩-玻尔兹曼定律

维恩位移定律：$T\lambda_m=b$.



普朗克公式：$R(\lambda,T)=\frac{2\pi hc^2}{\lambda^5(e^{hc/\lambda kT}-1)}$.

光电效应：$hv=K+\phi=\frac{1}{2}mv^2+\phi$

逸出光电子的最大动能

单个光子能量　金属逸出功

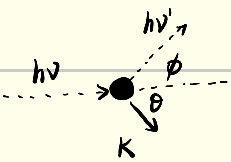
截止频率 v_0　$hv_0=\phi$

$K=hv-\phi=eV \Rightarrow V=\frac{hv}{e}-\frac{\phi}{e}$

遏止电压

康普顿效应：

$\lambda f=v$ ☆



$E=hv=\hbar\omega$；$p=\frac{h}{\lambda}=\hbar k$.

$h=2\pi\hbar$

$hv+m_0c^2=hv'+mc^2 \Rightarrow \frac{hc}{\lambda_0}=\frac{hc}{\lambda'}+m_0c^2\left(\frac{1}{\sqrt{1-(v/c)^2}}-1\right)$ —— ①

$\frac{h}{\lambda_0}=\frac{h}{\lambda'}\cos\phi+\frac{m_0v}{\sqrt{1-(v/c)^2}}\cos\theta$ —— ②

$\frac{h}{\lambda'}\sin\phi=\frac{m_0v}{\sqrt{1-(v/c)^2}}\sin\theta$ —— ③

$\Rightarrow \Delta\lambda=\lambda'-\lambda_0=\frac{h}{m_0c}(1-\cos\phi)$.

物质波（德布罗意波）：$E=hv$，$p=\frac{h}{\lambda}$

不确定性原理：$\Delta x\cdot\Delta p\ge\hbar/2$. $\Delta E\,\Delta t\ge\hbar/2$. $\frac{\partial^2 f(x)}{\partial t^2}=-\omega^2 f(x)$ 时间上.

薛定谔方程：从波动方程 $\frac{\partial^2 f(x)}{\partial x^2}=-k^2 f(x)$ 空间上 出发；$\frac{\partial^2\psi}{\partial x^2}=-k^2\psi$ 解形为 $\psi(x)=Ae^{i(kx-wt)}$

$p=\frac{h}{\lambda}=k\hbar\Rightarrow k=\frac{p}{\hbar}$ ∴ $\frac{\partial^2\psi}{\partial x^2}+\left(\frac{p}{\hbar}\right)^2\psi=0\Rightarrow p^2\psi=-\hbar^2\frac{\partial^2\psi}{\partial x^2}$

粒子总能量 $E=\frac{p^2}{2m}+V(x)$ $\Rightarrow p^2=2mE-2mV(x)$ 代入⇒ $2m(E-V(x))\psi=-\hbar^2\frac{\partial^2\psi}{\partial x^2}$

动能 势能

$\Rightarrow -\frac{\hbar^2}{2m}\cdot\frac{\partial^2\psi(x)}{\partial x^2}+V(x)\psi(x)=E\psi(x)$

$\omega=\frac{E}{\hbar}$ 一维定态方程.

含时部分：$\phi(t)=e^{-i\omega t}=e^{-i\frac{E}{\hbar}t}$ $\psi(x,t)=\psi(x)\cdot e^{-i\frac{E}{\hbar}t}$

$\Rightarrow \frac{\partial\psi(x,t)}{\partial t}=\psi(x)\cdot\left(-i\frac{E}{\hbar}\cdot e^{-i\frac{E}{\hbar}t}\right)=-i\frac{E}{\hbar}\psi(x,t)$. ①

代入一维定态：$\left(-\frac{\hbar^2}{2m}\cdot\frac{\partial^2}{\partial x^2}+V(x)\right)\psi(x)\cdot e^{-i\frac{E}{\hbar}t}=E\psi(x)\cdot e^{-i\frac{E}{\hbar}t}$.

$\Rightarrow \left(-\frac{\hbar^2}{2m}\cdot\frac{\partial^2}{\partial x^2}+V(x)\right)\psi(x,t)=E\psi(x,t)$

由① ⇒ $E\psi(x,t)=-\frac{\hbar}{i}\cdot\frac{\partial\psi(x,t)}{\partial t}=i\hbar\frac{\partial\psi(x,t)}{\partial t}$

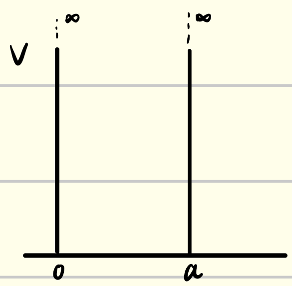
⇒ 一维含时方程：$i\hbar\frac{\partial\psi(x,t)}{\partial t}=\left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}+V(x,t)\right)\psi(x,t)$.

三维含时：$i\hbar\frac{\partial\psi(\vec r,t)}{\partial t}=\left(-\frac{\hbar^2}{2m}\nabla^2+V(\vec r,t)\right)\psi(\vec r,t)$.

三维定态：$\left(-\frac{\hbar^2}{2m}\nabla^2+V(\vec r)\right)\psi(\vec r)=E\psi(\vec r)$

$\psi^2(x)$是粒子出现在x处的概率密度 $\psi^2(x)=\psi^*(x)\cdot\psi(x)$ $\Rightarrow \iiint_V\psi^*\psi\,dV=1$ 空间中粒子出现全概率为1.

9 一维无限深势阱中的粒子



无限高的势壁

内部 $0<x<a$ 电势能$V(x)=0$

外部 $x\le 0$ 或 $x\ge a$时，$V(x)=\infty$

$0<x<a$：

由一维定态方程：$-\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2}+V(x)\right)\psi(x)=E\psi(x)\Rightarrow -\frac{\hbar^2}{2m}\cdot\frac{\partial^2\psi(x)}{\partial x^2}=E\psi(x)\Rightarrow\frac{\partial^2\psi(x)}{\partial x^2}+\frac{2m}{\hbar^2}E\psi(x)=0$

$\Rightarrow\frac{\partial^2\psi(x)}{\partial x^2}+k^2\psi(x)=0$ ∴ $\psi(x)=A\sin kx+B\cos kx$.

$\left(k=\frac{2mE}{\hbar^2}\right)$

在势阱壁：$\psi(0)=0$, $\psi(a)=0$ 此时 $\psi(x)=A\sin kx$

对于 $x=0$，始终 $\psi(x)=0$；$\psi(a)=0 \Rightarrow \sin ka=0$ $\therefore ka=n\pi \Rightarrow k=\frac{n\pi}{a}$, $n=1,2,3\cdots$

即 $\psi(x)=A\sin\frac{n\pi x}{a}$ $\therefore \int_0^a \psi^*(x)\psi(x)dx=1 \Rightarrow A=\sqrt{\frac{2}{a}}$ $\Rightarrow$ 粒子定态波函数 $\psi_n(x)=\sqrt{\frac{2}{a}}\sin\frac{n\pi}{a}x$, $n=1,2,3\cdots$

代入定态波函数到 $V(x)=0$ 的一维定态方程解得能量：

$$E_n=\frac{n^2\pi^2\hbar^2}{2ma^2} \quad n=1,2,3\cdots$$

$\Rightarrow$ 能量是量子化的

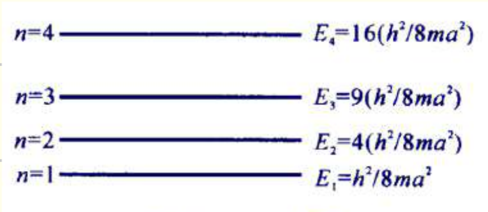


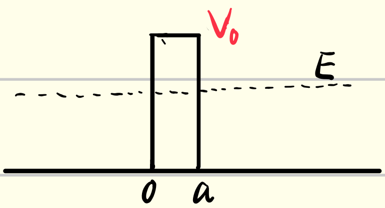
图 21.8 能级图

最小能量

$E=\frac{\pi^2\hbar^2}{2ma^2}$ 零点能

由 $k=\frac{n\pi}{a}=\frac{2\pi}{\lambda_n}$ $\Rightarrow$ 驻波波长：$\lambda_n=\frac{2a}{n}$ $n=1,2,3\cdots$

势垒 & 隧道效应



势能函数 $V(x)=\begin{cases} V_0, & 0\leq x\leq a \\ 0, & x<0, x>a \end{cases}$

一维定态：$E\psi(x)=\left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}+V(x)\right)\psi(x)$

$$\Rightarrow \frac{\partial^2\psi(x)}{\partial x^2}+\frac{2m}{\hbar^2}(E-V(x))\psi(x)=0$$

$x<0$, $x>a$ 时，$k_1=\frac{1}{\hbar}\sqrt{2mE}$ ；$0\leq x\leq a$ 时，$k_2=\frac{1}{\hbar}\sqrt{2m(E-V_0)}$

$\therefore$ $x<0$ 时，$\psi_1(x)=A_1e^{ik_1x}+A_2e^{-ik_1x}$

$0<x<a$ 时，$\psi_2(x)=B_1e^{ik_2x}+B_2e^{-ik_2x}$ $\Rightarrow$ 连续：$\begin{cases} \psi_1(0)=\psi_2(0) \\ \psi_2(a)=\psi_3(a) \end{cases}$

$x>a$ 时，$\psi_3(x)=Ce^{ik_1x}$

从 $x<0$ 到 $x>a$ 的概率，用透射率 T 表示，定义为透射波与入射波强度之比

$$\Rightarrow T=\frac{|C|^2}{|A_1|^2}\propto e^{-\frac{2}{\hbar}\sqrt{2m(V_0-E)}x}=e^{-2kx}$$

$$k=\frac{1}{\hbar}\sqrt{2m(V_0-E)}$$

6. 氢原子结构：

玻尔理论：原子存在系列确定能量的稳定状态.(定态)

处于定态时，电子稳定圆周：角动量 $L=mvr=n\hbar$ $n=1,2,3\cdots$

轨道半径量子化：$\frac{1}{4\pi\varepsilon_0}\cdot\frac{e^2}{r^2}=m\frac{v^2}{r}$ $\Rightarrow r_n=n^2\frac{\varepsilon_0 h^2}{\pi me^2}$ r_1为玻尔半径 $r_n=n^2r_1$

能量：$E_n=\frac{1}{2}mv_n^2-\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r_n}=-\frac{1}{n^2}\left(\frac{me^4}{8\varepsilon_0^2h^2}\right)$ $n=1,2,3\cdots$ $\Rightarrow E_n=\frac{E_1}{n^2}$

· 氢原子光谱：$\underset{i}{\text{高能级}}\xrightarrow{\text{跃迁}}\underset{f}{\text{低能级}}$，发射光子 $E_i-E_f=h\nu=\frac{hc}{\lambda}$

$\frac{1}{\lambda}=R\left(\frac{1}{n_f^2}-\frac{1}{n_i^2}\right)$ 里德伯常数.